Misc. on Transport

① Runaways → Strong E case

→ message is that stationary state not always possible for strong perturbation

→ basic idea:

- consider electron at $v \lesssim v_{th}$

- in one mean free time, mft, increment in speed is:

$$\delta v \sim \frac{qE}{m_e} \tau_{mfp} \sim \frac{qE}{m_e} \frac{l_{mfp}}{v_{th}}$$

$$\sim \frac{qE}{m_e} \left(\frac{1}{n\sigma v_{th}} \right)$$

but $\boxed{\sigma \sim 4\pi (e^2)^2 \ln\Lambda / m_e^2 v_{th}^4}$

$(\sim \pi b^2 \ln\Lambda)$

so

$$\boxed{\delta v \sim \frac{v_{The}^3 \, m E}{4\pi e^3 \ln\Lambda \, n_0}}$$

$$\boxed{\frac{\delta V}{V_{Th}} \sim V_{Th}^2 m_e E \Big/ 4\pi e^3 \ln\Lambda\, n_0}$$

$\delta V / V_{Th} \sim 1 \Rightarrow V_{crit}$ defined. → critical velocity for run-aways.

$\Rightarrow E_{crit}$ "

$$V_{crit} \sim \left(\frac{n e^3 \ln\Lambda}{m^2 E}\right)^{1/2}$$

critical field for $\delta V_{accel} \geq V_{The}$

$$\boxed{E_{crit} \simeq \frac{4\pi e^3 \ln\Lambda\, n_e}{T_e}}$$

→ critical electric field (Dreicer Field) for $\delta V \sim V_{Th}$

Now,

and for $\delta V > V_{Th}$, δV replaces V_{Th} in cross-section, etc.

so → momentum increment:

$$\Delta p \sim eE \frac{l_{mfp}}{V_{Th}} \to \frac{eE}{\delta V\, n\sigma(\delta V)}$$

$\Rightarrow$ $\underline{E} > \underline{E}_{crit}$

$$\Delta p \sim \frac{eE}{\delta v\, n\, \sigma_e(\delta v)}$$

$$\sim \frac{eE \quad m^2 (\delta v)^4}{\delta v \; n \; \ln\Lambda (e^2)^2 n_0}$$

$$\sim \frac{eE\, m^2 \delta v^3}{n \ln\Lambda\, e^3 n_0}$$

$$\sim m\, \delta v \left(\frac{\delta v}{v_{crit}}\right)^2$$

[momentum increment rises.

$\rightarrow$ $\delta v \sim v_{crit}$, $\underline{\Delta p \sim m\, \delta v}$

$\rightarrow$ [electrons accelerated without limit, if speed, E high enough

$$\Rightarrow \boxed{E > E_{crit} = 4\pi e^3 \ln\Lambda\, n_0 / T_0}$$

~ bulk "runs away"

$\Rightarrow$ Dreicer field for runaway.

→ $E < E_{crit}$ ⇒ tail runs away.

→ Time scales

$\dfrac{\tau_{ii}}{\tau_{ei,\,ee}} \sim \sqrt{m_i/m_e}$

$\tau^{E}_{e,i} \sim \dfrac{m_e}{m_i}$ ⇒ equilibration time longest.

$\dfrac{d}{dt} T_e = -\dfrac{1}{\tau^{E}_{e,i}} (T_e - T_i)$

$\tau_{ee,\,ei} < \tau_{ii} < \tau^{E}_{e,i}$

(b.) Applications – Dynamic Screening – $\left\{\begin{array}{l}\text{Collective Enhancement} \\ \text{of Collisional} \\ \text{Relaxation}\end{array}\right.$

Consider form $B_{\alpha\beta}$:

$$B_{\alpha\beta} = 2(e e')^2 \int_{-\infty}^{+\infty}\int_{k<k_{max}} \delta(\omega - \underline{k}\cdot\underline{v})\,\delta(\omega-\underline{k}\cdot\underline{v}')\, \frac{k_\alpha k_\beta\, d^3k\, d\omega}{k^4|\epsilon(\underline{k},\omega)|^2}$$

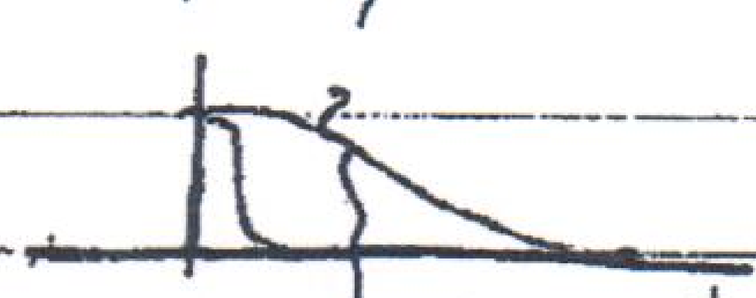
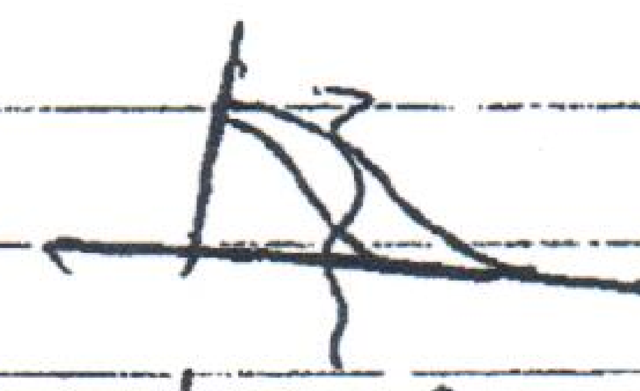
i.e. $\underline{k}\cdot\underline{v} = \underline{k}\cdot\underline{v}' \Rightarrow \underline{k}\cdot(\underline{v}-\underline{v}') = 0$

Consider stable, 2-species plasma. Then, have two collective resonances (i.e. weakly damped waves):
(no shift)

① → electron plasma waves; $\omega/k > v_{Te}$

② → ion acoustic waves; $v_{Ti} < \frac{\omega}{k} < v_{Te}$
(no shift fe)

① → tail of $\langle f\rangle_e$ ⇒ relatively few particles, little role in collision dynamics

② → if $T_e \gg T_i$  not 

weak damping $\langle f\rangle$ small — heavy ion, electron damping

∴ for $T_e \gg T_i$, ion acoustic resonance may enhance collisional relaxation (weakly damped modes)

To show, exploit 'pole approximation':

(Collective resonance enhancement of B)

$$\frac{1}{|\epsilon|^2} = \frac{1}{|\epsilon_r|^2+|\epsilon_{IM}|^2}$$

$$\cong \frac{1}{\left[\left((\omega-\omega_k)\frac{\partial\epsilon}{\partial\omega}\Big|_{\omega_k}\right)^2+|\epsilon_{IM}|^2\right]}$$

(damping $\rightarrow$ resonance linewidth)

$$\cong \frac{\pi}{|\epsilon_{IM}|}\underbrace{\delta(\epsilon_r)}_{\text{wave resonance}}$$

i.e. $\begin{cases} \epsilon_r = 0 \rightarrow \text{resonance location} \\ \epsilon_{IM} \rightarrow \text{resonance size/width} \end{cases}$

and note for electron-electron collisions:

$\omega \ll \underline{k}\cdot\underline{v},\ \underline{k}\cdot\underline{v}'$; due $\omega < \underline{k}\cdot\underline{v}_{Te}$ ordering

$\Rightarrow$

$$B_{\alpha\beta} \cong 2\pi e^4 \int_{-\infty}^{\infty}\int_{-\infty}^{+\infty} \delta(\underline{k}\cdot\underline{v})\,\delta(\underline{k}\cdot\underline{v}')\,\delta(\epsilon_r)\,\frac{k_\alpha k_\beta}{|\epsilon_{IM}|}\,d^3k\,d\omega$$

Change variables:

$K = \underline{k}\cdot\hat{n}$ (kappa) $\hat{n}$ unit along $\underline{v}\times\underline{v}'$

$k_1 = \underline{k}\cdot\underline{v}$

$k_2 = \underline{k}\cdot\underline{v}'$

then: $d^3k = dK\,dk_1\,dk_2/|\underline{v}\times\underline{v}'|$

so

$\Rightarrow$

$$B_{\alpha\beta} = \frac{2\pi e^4 n_\alpha n_\beta}{|\underline{v}\times\underline{v}'|} \, 2\int_{k>0} dk \int_{-\infty}^{+\infty} d\omega \, \frac{\delta(\epsilon_r(k,\omega))}{k^2 |\epsilon_{IM}|}$$

i.e. collapse k_1, k_2 integrals

Now, $\epsilon_r = 1 - \frac{\omega_{pi}^2}{\omega^2} + \frac{1}{k^2 \lambda_D^2}$

$\omega = kc_s / (1+k^2\lambda_{De}^2)^{1/2}$

ion-acoustic wave

$\rightarrow$ $$\epsilon_{IM} = \sqrt{\frac{\pi}{2}} \frac{\omega}{k^3} \left(\frac{1}{\lambda_{De}^2 v_{Te}} + \frac{1}{\lambda_{Di}^2 v_{Ti}} e^{-\omega^2/2k^2 v_{Ti}^2} \right)$$

electron L.D. $\omega \ll k v_{Te}$ — I.L.D

dominant contribution from shortwavelength ($k\lambda_{De} > 1$)

$\rightarrow$ (i.e. $\underline{\max}$ $1/|\epsilon_{IM}|$)

$\therefore$ $$\begin{cases} \omega \cong \omega_{pi} \\ \epsilon_{IM} = \sqrt{\frac{\pi}{2}} \frac{\omega_{pi}}{k^3} \left(\frac{1}{\lambda_{De}^2 v_{Te}} + \frac{1}{\lambda_{Di}^2 v_{Ti}} e^{-1/k^2\lambda_{Di}^2} \right) \end{cases}$$

$\Rightarrow$

$$\delta(\epsilon_r) = \delta\left(1 - \omega_{pi}^2/\omega^2\right) \cong \frac{1}{2} \omega_{pi} \left[\delta(\omega-\omega_{pi}) + \delta(\omega+\omega_{pi}) \right]$$

$$\Rightarrow \boxed{B_{\alpha\beta} = \frac{4\pi e^4 \omega_{pi}\, n_\alpha n_\beta}{|\underline{v}\times\underline{v}'|}\int \frac{dk}{k^3\,|\epsilon_{Im}(\omega_{pi},k)|}}$$

$$\varepsilon = k^2\lambda_{Di}^2$$

$$\epsilon_{Im} = \sqrt{\frac{\pi}{2}}\,\frac{\omega}{k^3}\left\{\frac{1}{\lambda_{De}^2 v_{Te}} + \frac{1}{\lambda_{Di}^2 v_{Ti}}\, e^{-\omega_{pi}^2/2k^2v_{Ti}^2}\right\}$$

$$\therefore \left\{\begin{array}{l} B_{\alpha\beta} = n_\alpha n_\beta \dfrac{2\sqrt{2\pi}\, e^4 v_{Te}\lambda_{De}^2}{|\underline{v}\times\underline{v}'|\ \lambda_{Di}^2}\displaystyle\int d\varepsilon \Big/ \left[1+\exp\left(-\frac{1}{2\varepsilon}+\frac{L_1}{2}\right)\right] \\ L_1 = \ln\left(\left(T_e/T_i\right)\left(v_{Te}^2/v_{Ti}^2\right)\right) \end{array}\right.$$

Now:

(i) $v_{Ti} < \frac{\omega}{k} < v_{Te} \Rightarrow \overset{(\omega_{pi}\lambda_{Di}/\omega_{pe}\lambda_{De})^2}{\cancel{1}} < \varepsilon < 1$

(ii) $L_1 \gg 1 \Rightarrow$ expand $O(1/L_1)$

i.e. dominant contribution when:

$\exp\left(-1/2\varepsilon + \frac{L_1}{2}\right) \ll 1 \Rightarrow \varepsilon \leqq 1/L_1$

note: $[1+\exp(-1/2\varepsilon + L_1/2)] \overset{\sim 1}{\equiv}$ i.e. denominator recall:

$$\boxed{B_{\alpha\beta} = n_\alpha n_\beta \left[\frac{2\sqrt{2\pi}\, e^4 V_{T_0}\, \lambda_{De}^2}{|\underline{V}\times\underline{V}'| \cdot \lambda_{D_i}^2}\right](1/L)}$$

Now:

above ($V \ll V_{Te}$)   ($V \sim V_{Te}$)

$$B_{\alpha\beta} = \underbrace{B_{\alpha\beta}^{\text{collective}}}_{\text{(collective resonance dominant)}} + \underbrace{B_{\alpha\beta}^{\text{coulomb}}}_{\text{(ballistic spectrum dominant)}}$$

as peaks in spectrum not coincident

$$B_{\alpha\beta}^{\text{coulomb}} = \frac{2\pi e^4}{|V-V'|} L \left[\delta_{\alpha\beta} - \frac{(V_\alpha - V_\alpha')(V_\beta - V_\beta')}{|V-V'|^2}\right]$$

$$\cong \frac{2\pi e^4}{V_{Te}} L_{\text{coulomb}}$$

$B^{\text{collective}} \geq B^{\text{coulomb}}$ if:

$$\boxed{\frac{T_e}{T_i L_1} \geq \underbrace{L_{\text{coulomb}}}_{\text{Coulomb log.}}}$$

Criterion for dominance of collective effect enhanced scattering (need $T_e \gg T_i$)